Derivation of Steering System Resonance Frequency for Steering Torque Response Design

- High-Accuracy Modeling of Dynamic Response as a Component of Steering Feel -

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According to Newton's second law, the steering wheel angle δ responds to an applied steering torque T_s by generating an angular acceleration, which is integrated into angular velocity and further integrated into the steering angle. This dynamic behavior appears as a resonance in the steering system, and this resonance determines the upper limit of the system's dynamic response. The higher this frequency, the more accurately and easily the steering angle can be controlled. These dynamics are represented by the combination of the steering system inertia moment Is shown in Fig. 1 and the planar motion model shown in Fig. 2.

The characteristic equation becomes a fourth-order polynomial in the Laplace variable s, making it difficult to derive closed-form expressions for the natural frequencies. The author approximately factorized this characteristic equation to obtain a concise formula. As shown in Fig. 3, the approximation accuracy of the steering system natural frequency ω_S and the planar motion system natural frequency ω_b is high.

In this study, an even more accurate formulation is derived. The key idea comes from the formulation of the two-degree-of-freedom resonance frequencies of the quarter-vehicle model shown in Fig. 4A. For the formulation of the lower resonance frequency, it is assumed that the other subsystem is stationary, as illustrated in Fig. 4B. For the higher resonance frequency, it is assumed that the other subsystem does not exist, as shown in Fig. 4C.

Following Fig. 4B, in deriving ω_s, it is assumed that the vehicle is laterally stationary (i.e., straight-ahead driving). Following Fig. 4C, in deriving ω_b it is assumed that the steering system does not exist. The expressions for ωs and ω_b obtained from these assumptions do not include vehicle speed. Therefore, as shown in Fig. 3, these frequencies are almost independent of vehicle speed. This implies that the values of ω_s and ω_b obtained under a certain vehicle speed remain valid for other speeds as well. By assuming an infinite vehicle speed and applying a Maclaurin expansion with respect to the nondimensionalized Is, the following equation is obtained:

$$\omega_S \approx \sqrt{(1 - I_{SN})\frac{C_f}{I_{SN} k_N{}^2 l} - I_{SN}\frac{C_r}{k_N{}^2 l}} \tag{1}$$

As shown in Fig. 5, this equation provides improved accuracy compared to the conventional formula.

By using this equation, the author believes that it is now possible to design response characteristics under force control with higher accuracy, supported by both mechanical and algebraic justification.

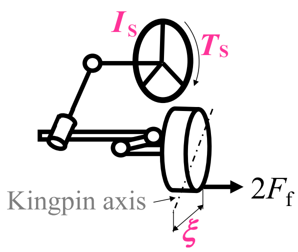

Fig.1 Model of steering system

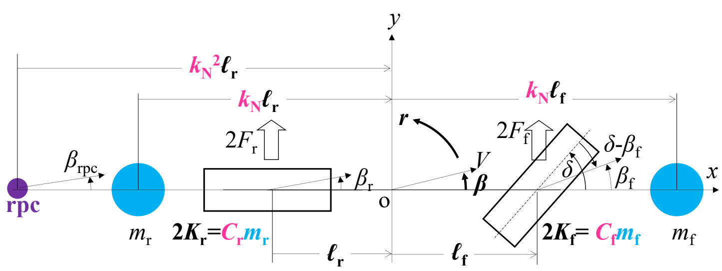

Fig.2 Model of planar system

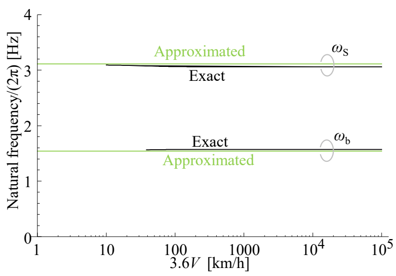

Fig.4 Accuracies of the conventional formulas

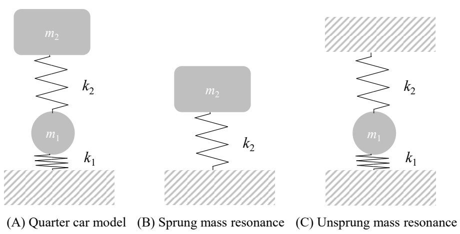

Fig.5 Quatoer- car model

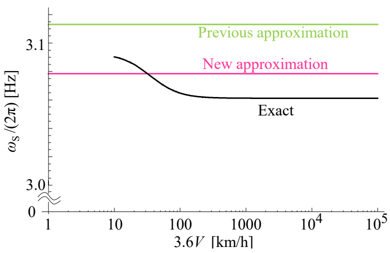

Fig.6 Accuracy of the new formula

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