

Bayesian Statistical Modeling of Corrosion Depth Evolution in Automotive Sheet-Metal Components Using Field-Return Data

Atsushi Tokunaga¹⁾

1) SUBARU Corporation, 1-1 Subaru-cho, Ota, Gumma 373-8555, Japan

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In automotive development, there are situations in which it is desirable to refer to the temporal evolution of corrosion depth, defined as the loss of base material thickness due to corrosion, in sheet-metal components such as body and suspension parts. One possible approach is to collect components from used vehicles that have been in service for several to more than ten years after their initial sale, measure their corrosion depths, and investigate the relationship between elapsed time since the vehicle was sold and corrosion depth by means of regression analysis. However, because the available data are limited in quantity and exhibit substantial variability, it is difficult to accurately assess the estimated corrosion depth and its associated uncertainty obtained from such analyses. In this study, we investigated a method for estimating the temporal evolution of corrosion depth as accurately as possible using regression analysis based on a Bayesian statistical model with strongly constraining subjective prior assumptions.

The random variable y , representing corrosion depth, was assumed to follow a Gamma distribution.

$$p(y) = \frac{1}{\beta^\alpha \Gamma(\alpha)} y^{\alpha-1}, \quad y \geq 0, \quad \mu = \alpha\beta$$

Where α and β are parameters. The μ is the mean of $p(y)$. The link function and predictor describing the time evolution of $p(y)$ were

$$\mu = w_1 [\{\log(1 + e^{k(x-w_2)}) - \log(1 + e^{-kw_2})\} / k]^{w_3}$$

Where w_i ($i = 1, 2, 3$) and k are parameters. The elapsed time x since the vehicle was sold is treated as an explanatory variable that does not assume that it follows a probability distribution. As the prior distribution for w_i ($i = 1, 2, 3$), we used the uniform distribution with a positive domain. The posterior distribution is estimated using the Markov Chain Monte Carlo (MCMC) method, and the Bayes predictive distribution is obtained by averaging over the posterior distributions.

$$p^*(y) = E[p(y|w_i, \alpha, \beta, k, x)]$$

Changes in the Bayes predictive distribution with respect to x represent the temporal evolution of corrosion depth. Using the 2.5th percentile $q_{0.025}(y)$ and 97.5th percentile $q_{0.975}(y)$ of the Bayes predictive distribution, we calculated an equal-tailed 95% Bayes prediction interval.

$$PI_{0.95} = [q_{0.025}, q_{0.975}]$$

The resulting set of values represents $q_{0.95}(y)$, whose 2.5th and 97.5th percentiles, denoted by $Q_{0.025}(q_{0.95})$ and $Q_{0.975}(q_{0.95})$, define an equal-tailed 95% Bayes credible interval, respectively.

$$CI_{0.95}(q_{0.95}) \approx [Q_{0.025}(q_{0.95}), Q_{0.975}(q_{0.95})]$$

The 100 synthetic datasets that mimic corrosion-depth and time data observed in suspension sheet-metal components collected from field vehicles were generated, and the performance of the Bayesian statistical model was evaluated using these datasets. Fig.1 shows an approximation to the 95% prediction interval $PI_{0.95}$ and a random sample from the true probability distribution for each dataset. In 95 out of 100 trials, random samples were contained within the 95% prediction intervals. Fig.2 shows an approximation to the 95% credible interval $CI_{0.95}(q_{0.95})$, estimated value, and true value for each dataset. In 94 out of 100 trials, the true values were contained within the 95% credible intervals. Since these proportions are close to 95%, the results suggest that the uncertainty associated with corrosion depth can be estimated using a Bayesian statistical model for real-world problems.

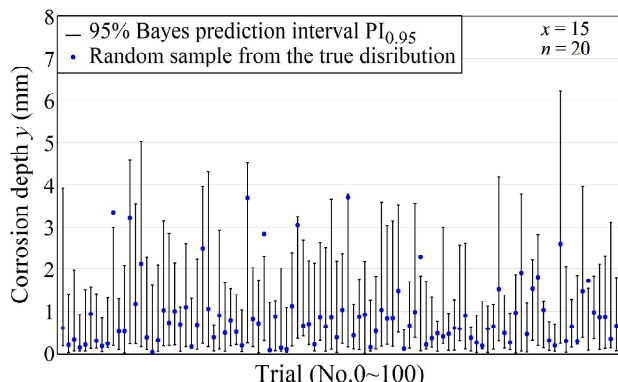


Fig.1 Evaluation results for the 95% Bayes prediction intervals over 100 trials at $x = 15$, sample size $n = 20$.

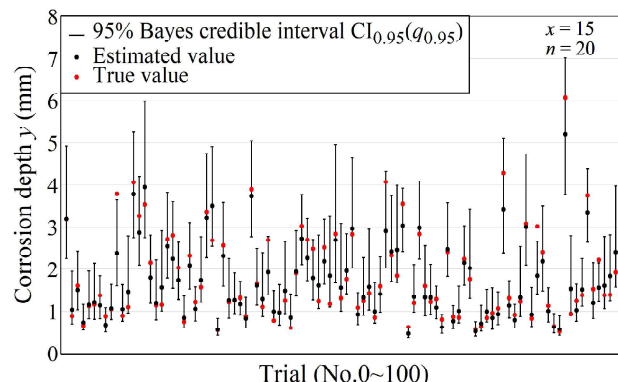


Fig.2 Evaluation results for the 95% Bayes credible intervals over 100 trials at $x = 15$, sample size $n = 20$.